Chapter 14. Transformities from Ecosystem Energy Webs with the Eigenvalue Method

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ABSTRACT

This paper describes a general procedure to compute solar transformities (ratios of emergy/energy) in ecosystems, applying the minimum-Eigenvalue method to previously published sets of data on energy flows. A tutorial with examples explains five steps: (a) drawing energy systems diagram with energy flows on the pathways, (b) writing emergy equations for each transformation, (c) entering energy flow values in a transformity-transformation matrix, (d) adding transformities to connect the data set to global solar emergy, and (e) using a specific computer software to calculate the transformities. Software templates are provided for making the calculation with MATHEMATICA and with TRUEBASIC. Transformities of biomass and other structural storages were computed from production data, whereas only the higher transformities for inter-compartment transfers were obtained for the energy networks without production data. Solar transformities obtained with these methods are included for Cone Spring (Iowa) groundwater meadow, an oyster reef, the Tabonuco rain forest, and dwarf cloud forest in the mountains of Puerto Rico.

INTRODUCTION

Transformity, the ratio of emergy/energy, measures the position of an energy flow or storage in the energy hierarchy of a network. This paper uses the minimum Eigenvalue method (Patterson, 1983; Collins and Odum, 2000) to determine transformities from published data sets. By making calculations in solar emergy units (solar transformities), networks of all kinds become comparable, even social networks. Solar transformities indicate the position of any energy flow or storage in the complex hierarchy of energy networks of the earth. Tables of solar transformities are also useful in emergy-emdollar evaluations of alternatives for development of the economy and conservation that recognize the work of nature. Based on our experience with these calculations, a tutorial is provided to help future evaluations.

E emergy is the energy of one kind previously used up directly and by indirect pathways in transformations required to generate a product or service. Transformity is the emergy per unit energy. Transformity increases with scale of time and space and marks the position of something in the universal hierarchy of energy that results when energy transformations are connected in series in a branching web with intermittent pathways, which is the way they are found in nature. See Odum, 1996, for concepts and examples.

The Patterson-Collins method of evaluating transformities from emergy equations was used to show the positions of species and other components of ecological systems in the universal energy hierarchy (Patterson, 1983, Collins and Odum, 2000). The program in MATHEMATICA was used for the matrix calculations as presented at the last conference (Collins and Odum, 2000). To make the application more portable, a program in TRUEBASIC (True BASIC, Inc., Hartford, Vermont) is also offered.
Energy network diagrams are usually highly aggregated simplifications with only a few pathways often arbitrarily selected based on the observer's concepts, available data, and scale of interest. The Patterson-Collins method sorts a set of emergy transformation equations without requiring a knowledge of all the pathways and branches. However, we recommend assembling the data with an energy systems diagram first to help avoid error and to ensure that each kind of energy flow is connected by at least one transformation pathway to the rest of the network.

Figure 1a is an example of a network of energy transformations which includes the several kinds of inter-unit connections: transformations in simple series, branches which are splits of the same kind of energy, branches which are two different kinds of energy, and transformations using two or more inputs. In order to express results in solar emergy units (solar emjoules), at least one energy transformation pathway is needed that connects solar emergy as in Figure 1a.
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OUTLINE OF THE METHOD: A PROCEDURE TUTORIAL

The following is a tutorial on using the Eigenvalue method of calculating transformities from data on energy transformations.

1. Assemble data by writing energy flow values on an energy network diagram. Figure 1 is a very simple example which has several energy transformations, one with a split of one kind of energy output, one with coproducts of two different kinds of energy output, and one with two inputs. The process of putting numbers on the pathways helps clarify which pathways are splits or junctions of the same kind of energy and which are different kinds of energy. For the purpose of these calculations, the energy systems diagram need not represent the whole ecosystem. Only the pathways with data need to be shown. However, for the calculations to succeed, every pathway with data needs to connect directly or indirectly with all the others. In other words, there must be no gap in the network.

In some networks, the energy flows are all consistent, referring to one ecosystem with values given in the same time units and referring to the same area. Such data can be represented with numbers on the pathways. However, the Eigenvalue method does not require that energy data in each transformation row be consistent. Where an energy transformation line (equation) is not part of the same ecosystem data, it is represented in the figure with an arrow along the path. See, for example, the microbe transformations in Figure 8.

2. Write emergy equations for each energy transformation. Emergy is conserved in energy transformations which do not involve feedbacks and loops. Each pathway of input emergy and output emergy is represented by the energy flow times the transformity. Transformities are the unknowns in this procedure and are designated by x’s (x1, x2, x3, etc.). An emergy equation has the emergy of one or more inputs (summed) set equal to emergy of one output. For example, in Figure 1:

\[ x5 \ast 0.2 + x6 \ast 0.1 = x7 \ast 0.01 \]
3. Next, prepare a table of these emergy equations with one line for each transformation in the system. For example, Figure 1b has the emergy equations for the system in Figure 1a. Each line in the list has the energy flows in and out of the transformation.

4. Next, put the input and output energy flows in a transformity-transformation matrix (table) in which transformities are the columns and designated with x’s (x1, x2, x3, etc.). Each line contains the one or more input energy flows with plus signs (no sign meaning “plus”), and the output energy flow is given a minus sign. Figure 1c is the transformity-transformation matrix for the system in Figure 1a.

5. If the data do not include an energy transformation row connecting solar emergy, add a line with the solar transformity from other sources of at least one of the kinds of energy already in the matrix. For example, suppose there is a compartment for plant biomass, but none for the solar emergy required. If
the solar transformity of plant biomass is known from other studies to be 2000 solar emjoules per one joule, add a line to the matrix with 2000 in the column x1 for solar emergy and -1 in the column for the biomass x2.

6. Next, load the software program to make the calculation (example, MATHEMATICA with its template in Figure 3). At the top of the calculation template, type in the values of the matrix from step #4. For example, the matrix of the simple example (Figure 1c) was entered in the template in Figure 3. Or make the calculations with TRUEBASIC, as described in the next section.

7. Run the program. For the MATHEMATICA template, hold down the SHIFT key and type ENTER. Figure 4 shows the result with a list of transformities at the bottom. The program will calculate the transformities, expressing them in units (emjoules) of the lowest quality energy in the matrix.

8. Running the program with the transformation matrix in Figure 2 yields the transformities in Figure 3 in solar emjoules/joule. Figure 4 contains another example of transformity calculations. The biomass diagrams of Figure 4a were connected to solar emergy with an assumed solar transformity of 10,000 sej/J (Figure 4d). Then the data were entered into the transformation-transformity matrix of the MATHEMATICA template. Figure 4e has the printout of the matrix and the resulting transformities.

It is not necessary to put all the data of an ecosystem into one large matrix. The solar transformity determined in one set of equations was used as a line item to another set of equations so that the second set could be linked. After transformities were tabulated, the positions of units in the systems diagrams can be adjusted so that items are in order of increasing transformity from left to right.

**CALCULATING EIGENVALUES WITH TRUEBASIC**

In the Appendix Table is the listing of program EIGNTRAN.tru, which makes the same matrix calculation as the MATHEMATICA template. TRUEBASIC can be obtained from True BASIC, Inc, 1523 Maple Street, Hartford, VT, 05047-0501. Phone 1-800-436-2111. http://www.truebasic.com.

Copy EIGNTRAN.tru to your hard disk. Load TRUEBASIC. Use its File Menu to open EIGNTRAN.tru. Enter the rows of the transformity-transformation matrix. Run the program.

**ECOSYSTEMS AND COMPARTMENTS**

Self organization appears to form systems of similar design as they join inputs from smaller scale and transformity to those of the next scale. The energy hierarchy in ecosystems has been represented with energy systems diagrams since 1966 (Watt, 1968; Holling, 1973). Symbols representing the parts and processes of ecosystems are arranged so that the turnover time, territory, and transformity increase from left to right (Figures 1-11). Values of energy flow are often represented on the pathways of these systems. Then transformities can be calculated for the flows with one of several methods. Common patterns of energy flow and transformation are observed in the self organization of ecological systems of many kinds. Partly this is due to real similarities and partly due to similar concepts used by people who assemble the network information. Larger organisms with larger areas of support and influence have higher transformities.

For any population or other aggregated compartment it is necessary to distinguish the smaller transformity of biomass production and storage from the higher transformity of the output transfer to other units in the network. All may be of interest. To explain the differences and ways of calculation, Figure 4 has energy flows and storages for one compartment box at steady state (inflows equal outflows and storage-structures constant). The word storage-structure refers to a compartment’s storage, which is
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Flow in Kilojoules per day =

(a)

Energy Transformation Process or Processes

Input 100

Used energy, not available 78

(b)

Storage-Structure 500

Production 10

Depreciation 5

Unassimilated 20

(c)

= Storage-structure in Kilojoules

Respiration = f + d

Figure 4. One compartment of an environmental system at steady state showing energy flows, storages, and concepts often used to represent structure and processes. Controlling inputs from other boxes are not included in this example. (a) Compartment without interior details; (b) compartment showing structural storage and related energy flows; (c) compartment simplified with values for production and storage only;
often a combination of functioning structure and stocks of stored substance.

Figure 4a represents an energy transformation where the details are aggregated. Many published studies of ecosystem networks use data aggregated so that the only flows given are inputs, outputs, and interunit transfers. When data are used from networks aggregated in this way, only the interunit flows are available to evaluate. A box compartment may be for a population of a species, but in other cases may be for a single organism or structure.

Other sets of data show more of the flows within each box compartment. Figure 4b has inputs, production of storage, storage, autocatalytic feedback, depreciation, and transfer to other compartments. Transformity of stored structures can be determined by multiplying inflow emergy by the turnover time and dividing by the stored energy. Because output transfers come from stored-structure, they represent the accumulation of more emergy and have higher transformities than the energy flows of productive input. Most data sets do not offer such details.

Figure 4c is a simplification of the numerical data to include inputs, output transfers, and only the production that generates the storage. The production numbers are written within the autocatalytic loop. If only the energy in the structural-storage is known, the production to storage can be estimated by dividing the storage by its turnover time (replacement time). For example, the production energy flow in Figure 4c could be obtained from Figure 4b by dividing the storage 500 J by the turnover time of 50 days to obtain 10 J/day.

Energy flow of production may also be calculated as the sum of the outflows from storage and the respiration. Respiration may be known or calculated from the storages using published data on metabolism per unit weight. In Figure 4c respiration is the sum of pathways feedback f and depreciation.
d. Energy flows may be estimated from data given in carbon use per time by multiplying by the energy/carbon ratio for that type of process.

From Figure 4e the transformity of the production to biomass storage was 100,000 sej/J. To verify the emergy of the biomass storage, multiply the input empower $1 \times 10^6$ sej/day by the turnover time of 50 days to obtain the biomass emergy storage = $5 \times 10^7$ sej. The transformity of the storage = emergy stored/energy stored = $(5 \times 10^7 \text{ sej})/(500 \text{ J}) = 100,000 \text{ sej/J}$ in agreement with the matrix calculation. The transformity of the biomass passed out to other units was larger, 500,000 sej/J.

The detritus concept has been used in ecology for a half century. Detritus is a pool of organic matter of many kinds and qualities with contributions from every level in biological food chains. Many ecosystem networks are drawn with a miscellaneous pool (the detritus) that receives leftovers from other compartments. Having a pool with organisms at many levels may increase system stability. Included are dead organisms, faecal pellets of unassimilated food, and organic secretions. For example, in Figure 4d, the unassimilated outflow is a different kind of energy from the transfer of storage-structure and has a different transformity. Flows into detritus might not be lumped together if the point of view of the analyst were on a smaller scale. Sometimes the bacterial and other living micro-organisms which constitute a

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**Figure 5.** Calculations using the energy systems diagram of Cone Spring ecosystem found in published literature. (a) Energy diagram and energy flows; (b) transformity-transformation matrix; (c) transformities calculated.
good part of detritus are grouped separately from the dead organic matter.

RESULTS

The transformity calculation procedures were used to evaluate other well known sets of energy data on ecosystems starting with Cone Spring and including a South Carolina Oyster reef, the tropical rain forest in Puerto Rico, and the Baltic Sea. For each set of ecosystem, data transformity computations were made and reported with a figure that includes the systems diagram, the energy flow values on the pathways or on adjacent arrows, the transformity-transformation matrix, and the resulting transformities.

Cone Spring

The Cone Spring system (Dames and Patten, 1981) has been used by many authors to introduce various energy network calculations and concepts (Wulff, Field, Mann and others, 1989). The system is an example of the coarse aggregation of Figure 4a. Figure 5 shows an energy systems diagram, transformity-transformation matrix, and resulting transformities. These transformities are for inter-unit transfers and not for the production or storages.

Note the higher transformity of the “bacteria” compared to the organic pool. Many networks
Figure 7. Energy systems diagram and transformity calculations for compartments of the Tabonuco rainforest in the Luquillo Mountains of Puerto Rico, with energy flows derived from Odum (1970 - Figure 2, page I-193). Heat sinks omitted.
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Figure 8. Energy transformations and transformities of microbes in the Tabonuco rain forest. Heat sinks omitted.

are aggregated so that the bacteria are regarded as part of the detritus, but the Cone Spring data reflect an attempt towards separation. Here the name bacteria is applied to the large scale cluster of microbiological processes, not to individual bacteria which are on a smaller scale. Aggregating small fast turnover items into clusters which have longer existence and larger scale increases the transformity. People studying ecosystems on a macroscale often aggregate the small components into groupings that may be real, but confusion is generated if these larger entities are named by the tiny items on smaller scale within.

Oyster Reef

Another much studied energy network is the South Carolina intertidal oyster reef depicted in Figure 6 (Dame and Patten, 1981). In the first line, the input of phytoplankton and particulate organics was given a solar transformity of 50,000 sej/J, a plausible value based on some other studies, so that the matrix could be based on solar emergy. High values were obtained for the animal categories, as might be expected where there was a high concentration of biomass.

Tabonuco Rainforest in Puerto Rico

Transformities were calculated for several sets of energy flow data for the mid altitude rain forests of the Tabonuco type in the Luquillo Mountains of eastern Puerto Rico. Energy flows in main categories of forest production in the Tabonuco forest at mid elevation were obtained from Odum (1970) with the results in Figure 7. Here, estimates of production were used to obtain transformities of the structural
storages (i.e. leaves, branches, roots, etc.), whereas only the transfer flows were obtained from the Cone Spring and Oyster data sets.

Data on microbial processes in the Tabonuco forest were evaluated in Figure 8, even though a microbial network model with energy flows was not available. Three fragments of information on the energy transformations leading to microorganisms were combined in a matrix calculation to obtain a transformity for bacteria and one for fungi, both considered as aggregated categories.

In Figure 9, fragments of energy transformation data on several animals in the Tabonuco forest were assembled in a transformation-transformity matrix and transformities calculated. Energy flows for several animals were obtained from Reagan and Waide (1996).

**Elfin Cloud Forest of the Luquillo Mountains**

A cloud forest of dwarf trees covered with epiphytes with dense mats of roots is found on mountain tops above 1000 m in the Luquillo rain forest. Almost continuously in the clouds, transpiration and solar insolation are both small. Data on the Elfin Cloud Forest of the mountain top from Weaver and Murphy (1990) were analyzed in Figure 10. Very high transformities were found, consistent with the very slow growth rates known from many studies there.

**SUMMARY**

The examples in this paper show how the Eigenvalue method can estimate transformities of systems by combining data from different sources with previously determined transformities. Transformities of interpopulation flows result from typical network data, but transformities for the
populations can be obtained where there are energy transformation data of production and growth. The method is synthetic when it describes the energy transformation hierarchy of a system with a set of transformities. With or without preconceived ideas about network connections, the hierarchy emerges from the energy transformation data of the parts.

ACKNOWLEDGMENT

This study is part of the Long Term Ecological Research Program 2001 on the tropical montane rainforest in Puerto Rico, coordinated by Prof. J. Zimmerman.

REFERENCES

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APPENDIX TABLE
Program EIGNTRAN.tru for Calculating Transformities from Energy Transformation Data

20 ! EIGNTRAN.tru
30 ! Dennis Collins, Dept of Mathematics, Univ of Puerto Rico, Mayaguez
40 OPTION BASE 0
110 ! Program in TRUEBASIC for Calculating Transformities from Energy Equations
120 ! Enter Transformation-transformity matrix as rows of DATA starting in line 4000
190 ! least squares min length solution of Ax = b
210 ! by singular value decomposition and generalized inverse
230 ! enter dimensions of augmented matrix (A b) on line 3000
250 ! enter augmented matrix (A b) row-by-row starting on line 3010
270 read m, n
290 let n = n - 1
310 ! n is number of columns of matrix A within program, so change
330 ! n = n - 1 in line 290
350 dim a(0,0)
360 mat redim a(m, n+1)
370 dim v(0,0)
380 mat redim v(n, n)
382 dim tv(0)
383 mat redim tv(n)
400 dim z(0)
410 mat redim z(n)
420 dim zl(0)
430 mat redim zl(n)
450 dim b(0,0)
460 mat redim b(n, n)
470 dim c(0,0)
480 mat redim c(m, n)
500 dim d(0,0)
510 mat redim d(n, m)
520 dim e(0,0)
530 mat redim e(n, m)
550 dim x(0)
560 mat redim x(n)
Appendix Table (continued)

580 read t
600 for i = 1 to m
620 for j = 1 to n+1
640 read a(i, j)
660 print a(i, j); “ “;
680 next j
700 print
720 next i
740 for i = 1 to n
760 for j = 1 to n
780 let v(i, j) = 0
800 next j
820 let v(i, i) = 1
840 next i
860 let c1 = n*(n-1)/2
880 for j = 1 to n-1
900 for k = j+1 to n
920 let p = 0
930 let q = 0
940 let r = 0
960 for i = 1 to m
980 let p = p+a(i, j)*a(i, k)
1000 let q = q+a(i, j)*a(i, j)
1020 let r = r+a(i, k)*a(i, k)
1040 next i
1060 if q >= r then goto 1150
1100 let c_t = 0
1110 let s = 1
1130 goto 1290
1150 if q*r = 0 then goto 1490
1190 if (p*p)/(q*r) < t then goto 1490
1230 let q = q-r
1240 let v_t = sqr(4*p*p+q*q)
1260 let c_t = sqr((v_t+q)/(2*v_t))
1270 let c_t = sqr((v_t+q)/(2*v_t))
1280 let s = p/(v_t*c_t)
1300 for i = 1 to m
1320 let r = a(i, j)
1340 let a(i, j) = r*c_t+a(i, k)*s
1360 next i
1380 for i = 1 to n
1400 let r = v(i, j)
1420 let v(i, j) = r*c_t+v(i, k)*s
1440 let v(i, k) = -r*s+v(i, k)*c_t
1460 next i
1470 goto 1510
1490 let c1 = c1-1
1510 next k
1530 next j
Appendix Table (continued)

1550 if c1 > 0 then goto 860
1590 for j = 1 to n
1610 let q = 0
1630 for i = 1 to m
1650 let q = q+a(i, j)*a(i, j)
1670 next i
1690 let q = sqr(q)
1700 let z(j) = q
1720 if q < t then goto 1820
1760 for i = 1 to m
1780 let a(i, j) = a(i, j)/q
1800 next i
1820 next j
1840 print
1860 print "matrix U with Ut*U = I"
1880 for i = 1 to m
1900 for j = 1 to n
1920 print a(i, j); "  ";
1940 next j
1960 print
1980 next i
2000 print
2020 print “singular values”
2040 for j = 1 to n
2060 print “z(“; j; “)= “; z(j)
2080 if abs(z(j)) < t then goto 2140
2120 let z1(j) = 1/z(j)
2140 next j
2160 print
2180 print “orthogonal matrix V”
2200 for i = 1 to n
2220 for j = 1 to n
2240 print v(i, j); "  ";
2260 next j
2280 print
2300 next i
2320 print
2321 let sv=v(1,n)
2322 for j=1 to n
2323 If abs(v(j,n))<abs(sv) then let sv=v(j,n)
2324 next j
2325 for j=1 to n
2326 let tv(j)=v(j,n)/sv
2327 print tv(j)
2328 next j
2329 print
2340 for i = 1 to n
2360 for j = 1 to n
2380 let b(i, j) = z(i)*v(j, i)
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Appendix Table (continued)

2400 next j
2420 next i
2440 print “check A= U*S*Vt”
2460 for i = 1 to m
2480 for j = 1 to n
2500 let c(i, j) = 0
2520 for k = 1 to n
2540 let c(i, j) = c(i, j)+a(i, k)*b(k, j)
2560 next k
2580 print c(i, j); “ “;
2600 next j
2620 print
2640 next i
2660 print
2680 for i = 1 to n
2700 for j = 1 to m
2720 let d(i, j) = z1(i)*a(j, i)
2740 next j
2760 next i
2780 print “generalized inverse= V*(S+)*Ut”
2800 for i = 1 to n
2820 for j = 1 to m
2840 let e(i, j) = 0
2860 for k = 1 to n
2880 let e(i, j) = e(i, j)+v(i, k)*d(k, j)
2900 next k
2920 print e(i, j); “ “;
2940 next j
2960 print
2980 next i
3000 print
3020 print “least squares min length solution”
3040 for i = 1 to n
3060 for j = 1 to m
3080 let x(i) = x(i)+e(i, j)*a(j, n+1)
3100 next j
3120 print “x(“; i; “)= “; x(i)
3140 next i
3150 ! dimensions of matrix next: rows, columns
3160 data 3, 4
3180 data .000001
4000 data 100,1,-11,0
4020 data 100,2,-21,0
4030 data 100,0,-1,0
5000 end
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