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Emergy and Efficiency Analysis of Historical Bubbles

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ABSTRACT

Human societies comprise energy-consuming beings, who are vast consumers of energy, as both individual organisms and organized societies. Hence, in principle, it should be possible to utilize thermodynamic approaches to generate models of historical phenomena. The existing energy-related efficiency-discounted exponential growth (EDEG) framework for historical resource bubbles and dynasties involving a nonrenewable resource is presented. As EDEG is presented, its relationship to emergy-related concepts and approaches are explored and analyzed. Finally, future research possibilities for expanding the physical, closed system framework behind EDEG into the open system emergy framework.

CONTEXT

A framework for a science of human history has been developed from a physics perspective utilizing no-renewable quantities and approximating historical dynasties as closed systems. However, it is important to also consider open systems and better constrain models with better physical data. The emergy approach offers both capabilities. This paper will explore emergy concepts as related to past work.

INTRODUCTION

Following the success of physical scientists such as Isaac Newton with the law of universal gravitation, it had been the dream of some social scientists to develop a science of society and history.

Coming at the top of a wave of scientific discoveries, and excited also by the political and social changes in the European world around him, he [father of Sociology August Comte] conceived the idea, which was shared in varying degrees by other thinkers of his time, that science might be unified and used in such a way as to harmonize the discords of human life (Marvin, 1936).

Unfortunately, the complexity of social phenomena has been overwhelming, and relatively little progress had been made.

However, as has been observed by physical and biological scientists, energy is a common thread. Humans and the societies that comprise them, are all energetic entities and the consumers of energy. Some aspects of a science of society and history could be constructed from energy principles and observations. We can go a step further. Humans consume energy (physics usage of term). Therefore, the laws of thermodynamics should be able to, in principle, constrain and drive a science of society and history.

This paper provides a framework for analyzing history by utilizing thermodynamics. It first applies this framework to social-economic structures involving physical resources such as gold and then to macro-historical structures such as dynasties. The physical and biological sciences can go deeper than mere, empirical computations by using characterizations and principles that allow for a unified framework of analysis. While work has already been conducted using physics terminology, this paper shall explore the application of emergy concepts to that past work.

Emergy is the availability of energy of one kind that is used up in transformations directly and indirectly to make a product or service. The unit of emergy is the emjoule, a unit referring to the available energy of one kind consumed in transformations. (Odum, Brown, Brandt-Williams, 2000).

The approaches have similarities, yet differ. The earlier, physical *entropic* approach has focused upon the consumption of nonrenewable quantities, while the *emergy* approach focuses on flows (albeit with stocks). The *entropic* approach attempts to characterize forces and tendencies, while the emergy approach focuses on energy concentration and usage. Physicists live in a universe of closed systems whereas the ecologists who formulated the emergy approach live in a universe of open systems. (These are the author's own interpretations.)

There are approximately 5000 years of recorded human history and long-term trends such as the overall exponential increase in human population and advances in technology. However, temporary deviations from trends are even more interesting. These deviations might be the product of regularly occurring tendencies, processes and structures. We see such processes and structures repeatedly arising in world history. Dynasties and empires come and go. Rise-and-fall patterns are also seen in economics and business. All of these patterns involve the build-up, release and flow of energy. Hence, in principle, the emergy involved in these processes should be determinable.

Conceptual Framework

A useful analogy is of water flowing down a river. Storm systems raise water up on to mountains, creating a potential. Gravity pulls the water down from mountain to sea. Sometimes the flow of water gets blocked from logjams and other causes. Complex river channels and other means emerge to breakup or overcome the barriers, and the force is relieved, restoring the system to a form of equilibrium.

An even more important flow is that of sunlight to the Earth's surface, and eventually back into space as radiated thermal energy. Although some light simply gets reflected back into space. Must of the remainder of that solar energy goes through complicated processes in which the quality of that energy degrades.

It has been observed that entropic potential results in the emergence of complex dissipative structures (Annila and Salthe 2010, Ciotola, 2001, Spier, 2009). Where a thermodynamic potential exists, there is substantial evidence that systems tend to configure themselves to maximize their rate of entropy increase. This tendency results in the emergence of structures that consume and dissipate entropic potentials. Atmospheric convection columns allow faster removal of thermal energy from the hot surface of the Earth into the coldness of space. Tornados and hurricanes are even more complex, faster mechanisms. The emergy approach explains such phenomena through the maximum power principle (Center for Environmental Policy, Energy, Exergy and Thermodynamics, 2018).

The further a system is removed from equilibrium, the more likely it is for complexity to form into mechanisms to consume potential (Prigogine, 1967). For example, convective energy transport structures (cells of hot plasma) form in stars as their temperature gradient becomes steeper (Carroll and Dale Ostlie, 2007).

If a complex structure can perform work, we can call it an *engine*. Steam engines in electric power plants are a type of engine called a *heat engine*. A heat engine *consumes* thermal potential. It *produces* work that can be used to operate machine tools, pump water or generate electricity. Such work can be called production. Only a portion of a *flow* of thermal energy can be used to generate work. This proportion is the engine's *efficiency*, which is the percentage of consumption is transformed into production. Hence, a heat engine also produces waste heat and entropy.

From an emergy perspective, efficiency is typically a comparison of the amount of solar energy equivalent (abbreviated sej) utilized to product a good via a particular process Some processes require more energy than others to produce the same amount of a product, such as in units of solar energy equivalent (sej)/unit of product. “For example, sunlight, fuel, electricity, and human service can be put on a common basis by expressing them all in the emjoules of solar energy that is required for each. In this case the value is a unit of solar emergy expressed in solar emjoules.” When that product is itself energy, then the units can be expressed as solar energy equivalent /Joule (sej/J) (Odum, Brown, Brandt-Williams, 2000).

A heat engine itself might not result in faster consumption of potential than a simple conductor. Yet what if the work from a heat engine is used to build additional heat engines? The result can be an exponential increase in the quantity (population) of heat engines. If each reproducing heat engine consumes potential, then there will be an exponential increase in the consumption of potential. Therefore, the emergence of reproducing heat engines is favored by the laws of thermodynamics.

The concept of an engine can be generalized to any mechanism that consumes potential and produces work. Bacteria consume high level energy and produce more bacteria, so they fit within this paradigm. Most living organisms do. Intelligent creatures can figure out additional means to consume thermodynamic potential, so intelligence is favored. The great organizations and technological developments of human civilization can consume potential even faster, through agriculture, trade, vast irrigation works, large coal mines, deep oil drilling and even the release of nuclear energy.

Limits and Decreasing Efficiency

There will always be factors that limit the growth in a system. A regime that experiences exponential growth will eventually begin to experience such limiting factors. Work on system dynamics such as the Club of Rome's *Limits to Growth* (Meadows, Meadows, Randers and Behrens, 1972) involves attempts to better understand these limiting factors. Such factors restrain growth and sometimes stop it altogether. Limiting factors usually exist due to a shortage of some essential resource or an excess of some "negative" resource. Turning to biology, an examination of reproducing cells shows that the chief limiting factors are typically a nutrient limitation or an accumulation of a toxic metabolite (Butler, 1996).

If there is a nonrenewable, built-up potential, such as oil, coal or gold, then generally, the intrinsic efficiency at which each additional unit of consumption is transformed into production decreases. This is expected: we go for the low hanging fruit first, then the slightly higher fruit, and only go for the hard-to-reach fruit at the top of the tree last. There are two chief types of efficiency: intrinsic efficiency and overall efficiency. Intrinsic efficiency is limited by the Second Law of Thermodynamics prohibition on entropy decreasing in an isolated system. Overall efficiency cannot exceed intrinsic efficiency, but it can be much lower. Technical improvements and economies of scale can help improve overall efficiency.

Overall efficiency functions can be in several forms, depending on the circumstances involved. For example, for a nonrenewable resource, the efficiency function may decay linearly or exponentially. There are ways of determining the overall efficiency function from actual data or known constraints.

APPLICATIONS, RESULTS AND ANALYSIS

The Rise and Fall of a Bubble

We have seen how the emergence of reproducing dissipative structures can lead to exponential growth. We have also seen how intrinsic efficiency decreases as a nonrenewable resource is consumed. Hence, we now have the conceptual means to understand the lifecycle of a bubble. To be clear, from this point further, a ***bubble*** will refer to *a rise-then-fall progression due to the emergence of reproducing, dissipative structures operating upon a limited, nonrenewable resource*.

1. A thermodynamic potential accumulates. In emergy analysis, this would be called a stock. For a stock to be a potential, relevant consumers or sinks must exist.

2. A reproducing, dissipative mechanism has emerged to consume the potential. In emergy analysis, this role could be fulfilled by a combination of producers and consumers.
3. The mechanism reproduces exponentially. Consumption increases exponentially.
4. Growth continues, but intrinsic efficiency decreases. In emergy analysis, growth is often represented by increased population(s) in a system.
5. Eventually either all the potential gets consumed, or the efficiency of exploiting it falls below the ability of the mechanism to maintain itself.
6. The progression of the bubble ends.

Since exponential growth is involved, but the transformation of consumption into production must be discounted by efficiency, this approach to bubble modeling is called *efficiency-discounted exponential growth (EDEG)*.

Application to Natural Resources

A bubble can involve physical or social resources. However, it is easier to begin analyzing physical resource bubbles, since there is often more available data and the examples are fairly straightforward. It is instructive to apply this bubble paradigm to a case involving natural resources such as the mining of precious metals (a stock). Mining is both a physical and social activity. Deposits of a commercially-demanded substance represent an economic potential (which is ultimately a thermodynamic potential). Mined substances are typically nonrenewable. If you extract a ton of that material, another ton does not form in the ground. Such substances took millions or even billions of years to form and accumulate. Therefore, there is only so much to extract: the mining bubble must eventually end. In other words:

past consumption + future consumption = constant

at any point of time. Further, since the resource is nonrenewable, the intrinsic efficiency of extracting the potential will tend to decrease over time (according the hypothesis above).

San Juan Mountain Area Mining Region

To apply bubble analysis of a mining region, it is best if the region considered is sufficiently large to initially support many actors, such as mining enterprises. Then in early stages, but after things get started, no individual mine can determine the fate of the entire region. For example, poor management at one mine is offset by effective management at another. The region should be sufficiently isolated so as not to be too influenced by external factors, but there must nevertheless be an internal or external demand for the substance.

The San Juan mountain area (known as the "San Juans") in southwest Colorado is such a region and is a suitable physical and social example of a bubble, including the mining society that developed in the region. The San Juans region of Colorado produced gold and silver (Henderson, 1926) from hundreds of mines, around which towns and communities eventually developed.

The San Juans were ruled by Spain until 1848, and then by the United States. Spanish gold mining of placer deposits took place between about 1765-1776. Some U.S. mining took place in 1860, but it was interrupted by U.S. Civil War. At this point, "only the smaller deposits of high-grade ore could be mined profitably." Mining slowly started again in 1869. There were 200 miners (producers) by 1870. An Indian Treaty was negotiated in 1873, which removed a major obstacle to an increase of mining (Smith, 1982 Twitty , 2010).

In 1881, a railroad service was established, resulting in a "decline in ore shipping rates." The regions heydays were between about 1889 and 1900. As the region matured, there was a major consolidation of mining operations as well as significant infrastructure improvements such as rail lines.

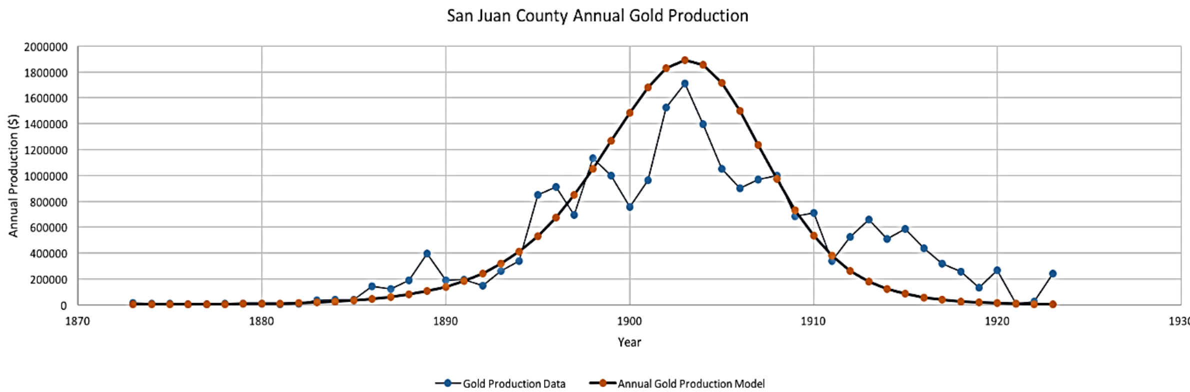

Figure 1. *Colorado San Juans gold production versus model (data: Henderson, 1926).*

By 1889, English investors had come to control the major mines. There were also labor troubles. The 1890 production total for San Juans was $1,120,000 in gold; $5,176,000 in silver. The region produced $4,325,000 in gold and $5,377,000 in silver in 1899 (Smith, 1982). Note that for mining, production refers to the processed metal rather than mere ore.

By 1900, the region began to take on more of the characteristics of a settled community. By 1909, "the gilt had eroded", dilapidation set in and the population decreased. World War I caused production to greatly fall, due to decreased demand from Europe (consumers), and the region lost workers. Farming became more important to local economy than mining. Silver and gold mining all but ceased by about 1921 (Smith, 2918).

An EDEG model was generated for the San Juans region and compared with actual mining data (Figure 1) (Ciotola, 2016). Exponential growth was initially facilitated by an increase in the quantity of mines, then later by the growing size of mines. Intrinsic efficiency decline in mining tends to comprise decreasing quality ore. For example, the early-mined placer deposits might comprise 100% gold, while late stage ore might comprise less than 1% gold. The model is somewhat higher than the peak, but parameters were adjusted to provide a better fit with the overall data. Deviations shown in the curve occurred due to both random events, social, economic and logistic "turbulence", business cycles and major external events. Transformity is not constant.

Emergence and Progression of a Single Dynasty

We will now express historical dynasties as emergent dissipative structures and generate power progression models of dynasties from fundamental principles. Here, the term *dynasty* is used broadly to refer to a continuous ruling group. It could be a related family but not necessarily so. In contrast, the term *society* will refer to a large group of related people, typically of a single or similar group of ethnicities, such as the Han people in China or the Frankish people in France. Dynasties exist within a society but can conquer other societies as well. For example, the society of Russian people produced a series of dynasties, and those dynasties sometimes conquered other societies.

Dynasties as emergent dissipative structures

Both physical and social built-up potential can drive the formation of dynasties. Within the context of a civilization progressing over centuries, it is often possible to degrade built-up potential even more quickly with high-level, governing social structures for a society. Hence, dynasties form to accelerate consumption of such potential. Hypothetically, dynasties should result in more rapid degradation of energy than does a more static society.

Each dynasty has a lifecycle. A dynasty is born, matures, endures awhile, then ends. A new dynasty will not necessarily follow an old one or might not immediately appear. Yet generally, dynasties continue to form, one after another, so long as there exists built-up potential that cannot be more quickly consumed by other means. The dynastic lifecycle can be described as a march towards equilibrium in terms of nonrenewable resources, and towards dynamic equilibrium in terms of on-going flows such as sunlight and rainfall.

We can consider large, independent, robust dynasties to be bubbles. A new dynasty within a society encounters a built-up potential of physical and social resources (e.g. goodwill), albeit of limited magnitude. The society governed by the dynasty fills the role of a collection of heat engines, producing both work and entropy. Prosperity expands exponentially, increasing the consumption of potential exponentially. Eventually, it becomes increasingly difficult for the dynasty to rely upon its store of physical and social resources, decreasing its efficiency. As efficiency decreases, the dynasty will experience social crises and will eventually stop functioning.

Considerations for modeling dynasties

It is simpler to model a sufficiently large, robust, independent dynasty than one that existed merely at the whim of its neighbors, for there are less significant dependencies, and thus it can be approximated as a substantially isolated system. We will examine Russia's Romanov dynasty as an example. Widely accepted start and end dates are 1613 and 1917 (Mazour and Peoples, 1975). Peter the Great and Catherine the Great were the two important rulers of the Romanov dynasty, and the Russian Empire gained much of its most valuable territory by the end of Catherine's reign in 1796. The Romanov dynasty was big, robust and essentially independent. It fought wars, but generally was not under serious threat of extinction. Even Napoleon could not conquer Russia.

By developing a fundamental approach to modeling the rise and fall of dynasties, it is possible to accept or reject models (within a range of uncertainty) based upon both qualitative historical evidence and quantitative historical data. We shall discuss generating models of the rise and fall of power of dynasties versus time and how a single dynasty can be modeled using the efficiency-discounted exponential growth (EDEG) approach (Ciotola, 2014). Such an EDEG model of a dynasty can be called a power progression (akin but not necessarily identical to an empower progression). The models shown should be considered mere first approximations rather than definitive assertions of fact. They are a beginning point of further explorations.

Historical dynasties are consumers of energy and producers of power, so models in terms of such quantities are inherently fundamental in that they can be derived directly from the laws of physics and expressed in physical quantities. Such models are not theories of everything, but rather describe certain aspects of broad macro-historical phenomena rather than the intricate workings of the interactions of individual people.

The term energy is meant in the physical sense here. There are several possible proxies of the physical energy of a dynasty, such as population governed or grain production. Each of these is translatable into physical units of energy. For example, the quantity of people multiplied by the mean Calorie diet per person will result in units of energy. These figures can be estimated for most dynasties over their lifespans, albeit with differing degrees of uncertainty.

Power is a physical term. It refers to energy expended per unit of time. Yet it also has meaning within social and political contexts, and will be discussed in both senses. Absolute power would generally be presented in physical units of power such as Watts. However, it is possible to express any type of power in terms of proportions, such as the ratio of power at a dynasty's peak to its start date. Such a ratio can apply to physical, political or even military power. Possibly, the EDEG approach can be utilized to model other types of power, such as political power. In fact, the EDEG approach provides a framework to explore the question of how political and physical power are related.

Exponential growth of dynasties

A new dynasty will tend to experience exponential growth. A chief characteristic of exponential growth is that growth feeds even more growth, resulting in an increasing rate of growth. Increases in population and power can become explosive. Nevertheless, the growth rate in early stages tends to be relatively flat, while the growth rate later tends to be relatively steep. The transition between "flat" and "steep" can be surprisingly sudden and disruptive (Meadows et al., 1972).

It will be assumed that dynasties will strive to grow exponentially. (This is a rebuttable presumption). If so, this certainly explains the rise of a dynasty. Sources of growth can include increased agricultural productivity, geographic expansion, and trade expansion.

The Romanov dynasty is shown with various growth rate models (Figure 2). The plot shapes appear similar, except that a greater rate produces a "sharper" corner. Also, notice the range of power values: a greater growth rate produces a disproportionately greater power value at later points of time. The growth rate function can be constrained by the data and an understanding of the growth mechanisms involved.

Limiting factors and decreasing efficiency

Another source of limiting factors is the increasing cost-per-unit (likely in terms of both money and emergy) to extract nonrenewable resources such as minerals. Societies attempt to use large-scale social and technical structures to shore up efficiency (e.g. San Juans mining case study), but these structures create additional challenges. Malthus (Heilbroner , 1980) pointed out limiting factors in the growth of agricultural production.

A dynasty will typically consume both conserved *and* renewable resources. Yet it is the consumption of one or more critical nonrenewable resources that determines the growth and decline characteristics of the regime. (Whereas the availability of renewable resources considered in emergy analysis may determine key characteristics of a society.) Production in a dynasty is ultimately dependent upon nonrenewable physical and social resources (otherwise dynasties would not typically end). Dynasties inevitably do end, which is typically preceded by a decline in power.

As the dynasty progresses, nonrenewable resources will be consumed, and efficiency will decrease. There will still be production until the end, but there will be a lower return on investment, so to speak. Physical causes of decay can include overuse of agricultural land leading to nutrient depletion, the build-up of toxins in the environment, and the depletion of old growth forests. Social causes can include running low on social goodwill, the increased dependency on expensive, monopolistic centralized institutions and structures, and the resulting decreased accountability of aristocratic “deadwood”. All such may have nonrenewable aspects.

The key impact of limiting factors, whether insufficient positive resources or excessive negative resources, is a decrease in the efficiency of whatever is acting as "heat engines" to do work. There are two types of decay, linear and exponential. Examples are compared (Figure 3). Note that efficiency here is shown as a proportion (multiply by 100 to get a percentage).

A linear approach is simpler to set up. Importantly, it also provides some reflection of overall efficiencies achieved through centralization and economies of scale as the dynasty progresses. Centralization can produce economies of scale that can boost net efficiency, but when a centralized system eventually goes bad, it can go bad fast! Failed central institutions can bring a dynasty crashing down quickly. This is an example of an irreversible process.

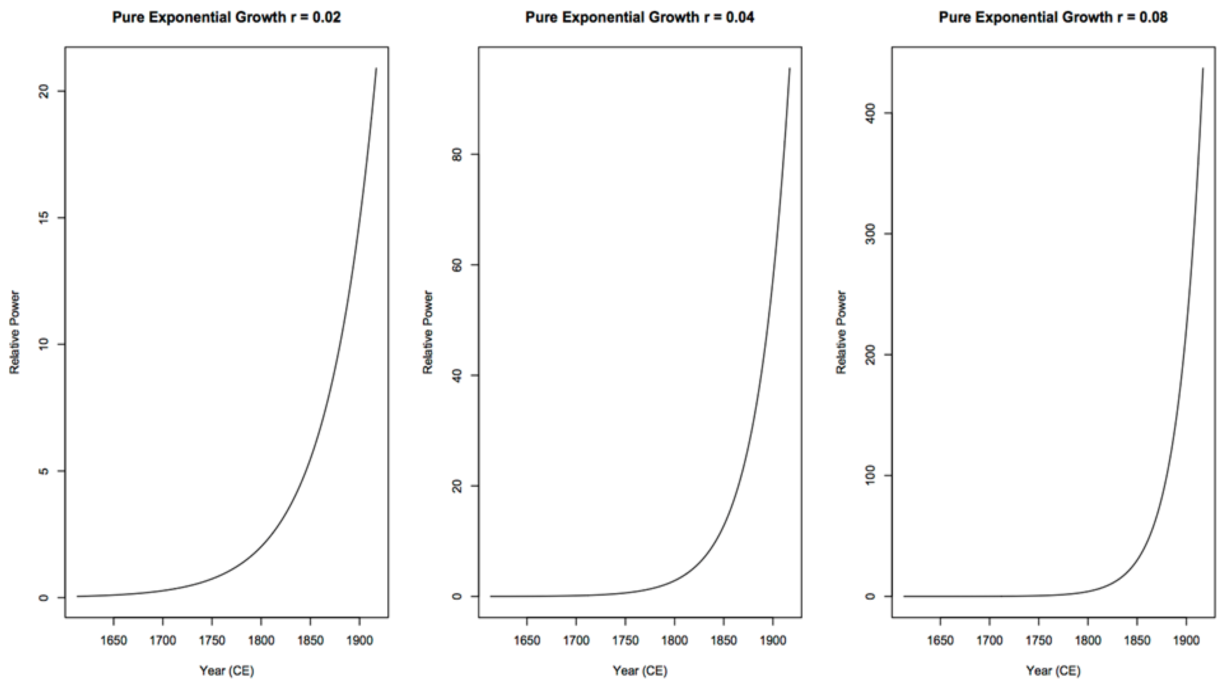

Figure 2. *Exponential growth for various growth rates.*

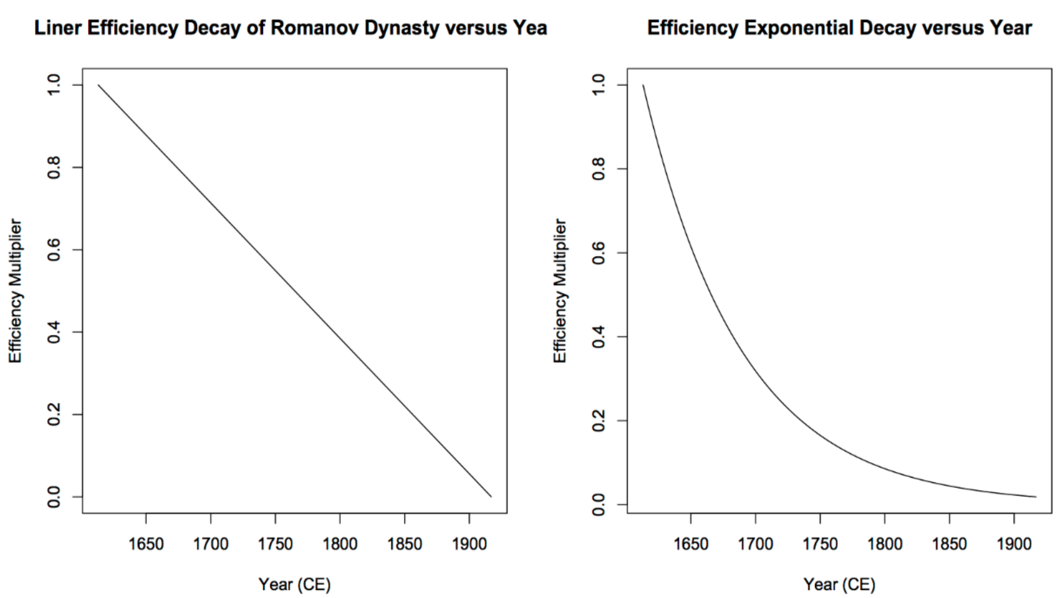

Figure 3. *Linear vs decay efficiency.*

A linear approach has unambiguous beginning and end points. Efficiency cannot be greater than 100% and is typically not lower than zero. Therefore, as a first approximation, one can set the overall efficiency to 100% at the start date of the dynasty and 0% at the end year (except that the math is simpler if the value 1 is used for 100%). Using a value of 0 for ending efficiency ensures that the dynasty ends by its historical end date. It is possible to use a value other than zero for the ending efficiency, but then some other factor must be used to end the dynasty. The following is an example of linear decay function:

efficiency = 1 - ((year - start year) / (end year - start year)).

As the year increases, efficiency will decrease. Using a lower initial efficiency reduces the magnitude of production increase for the dynasty compared to its initial production. It also flattens out the curve.

Generating a dynastic power progression

We now bring exponential growth and declining efficiency together (Figure 4). Growth will not only slow but often will start to reverse. Such growth and decline can be represented by an EDEG function, where the area under the curve represents either the total production or consumption of a conserved resource over time. The critical resource becomes more expensive as each successive unit of it is utilized. For example, decreasing soil nutrients and increased social overhead may result in less grain produced per hectare, which would correspond with both decreasing efficiency and increasing transformity). The following is an example of an EDEG equation:

y = efficiency function * exponential growth function.

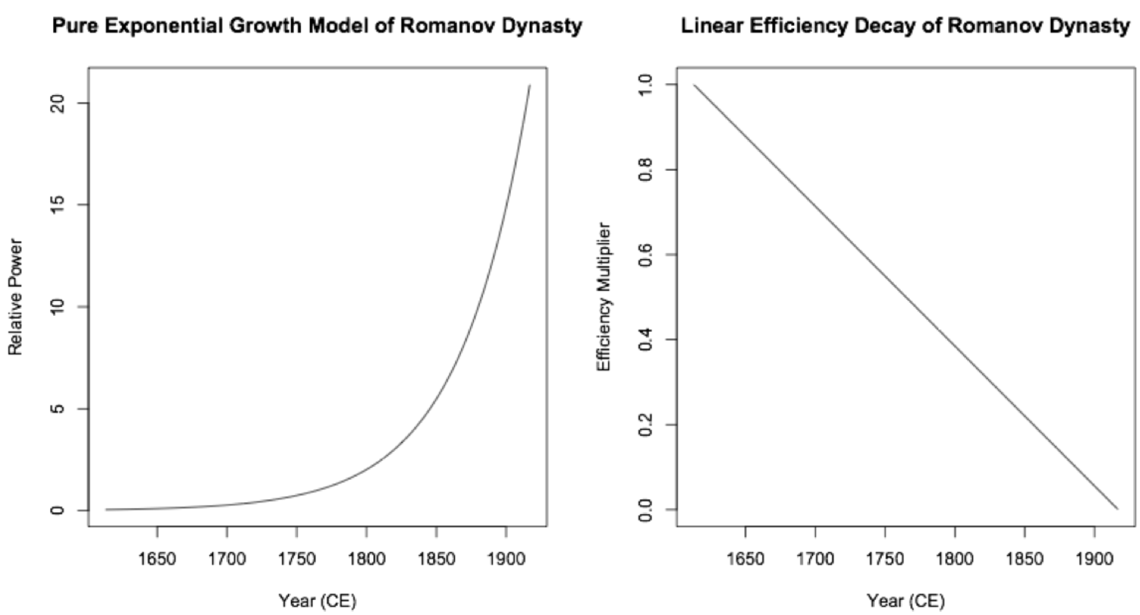

Figure 4. *Exponential growth and linear decay.*

Here is a simple way to generate a quantitative model for a dynasty. It is simplistic, but it generally produces qualitatively correct results. Assume exponential growth:

$$P_t = P_0\, e^{kt},$$

Where *P* is power, P_0 is initial power, *t* is time and *k* is a growth factor. Assume that a nonrenewable resource is being consumed that cannot be replaced within the lifetime of the dynasty. Then assume the efficiency of each subsequent unit of resource consumed produces power as a decreasing efficiency. Using the simple linear efficiency decay function from above:

$$\varepsilon = 1 - (\text{year - start year of dynasty}) / (\text{end year of dynasty - start year of dynasty}),$$

where ε is efficiency. Then the efficiency-discounted power is:

$$P = \varepsilon * P_0\, e^{kt}.$$

Substituting in our functions (utilizing linear decay):

$$Y = (1 - (\,(\text{year - start year}) / (\text{end year - start year})\,)\,) * P_0 * e^{k(\text{year - start year})}.$$

This produces a steady rise, a level period and a slightly faster decay. By discounting exponential growth by decreasing efficiency, we then have a rise and fall pattern that is consistent with the rise and fall of a dynasty.

Let us assume a conservative 1% growth rate for the Romanov dynasty. Let us further assume linear decay from 100% to 0% efficiency. A simulation has been written in the Ruby programming language. This language is mathematically robust, yet it involves code that is relatively easy to read and understand. The dynasty is run through the Ruby simulator, using the above parameters. The R language was utilized to generate a plot of the results (Figure 5).

Here the peak is close to 1820. Napoleon had been conquered, and the dynasty had achieved much of its geographic expansion by then. Yet by this time, social unrest began to shake the Romanov dynasty. Also, note how the dynasty power begins at a level of 1 and ends at a level of 0. This is conceptually appropriate, since the dynasty had to begin from something, but typically ends in nothing. (Actual power quantities can be used, but their explanation is beyond the scope of this paper). For example, the ancestors of the Romanovs existed before 1613, but the entire immediate family was killed during the Russian revolution. The peak occurs at a relative power value of height of 2.6, which indicates that the dynasty was over twice as powerful at its peak as it its beginning. Remember, this model is merely a hypothesis that is either valid or not for a particular level of uncertainty.

It may be of further interest to tie the rise and fall to patterns concerning the production and consumption of physical resources, to determine what correspondence, if any, there is between physical and political power. This can be explored by utilizing actual physical energy data (and emergy and transformity calculations) to produce a model of physical power and empower, and then compare such with evidence of political power over time.

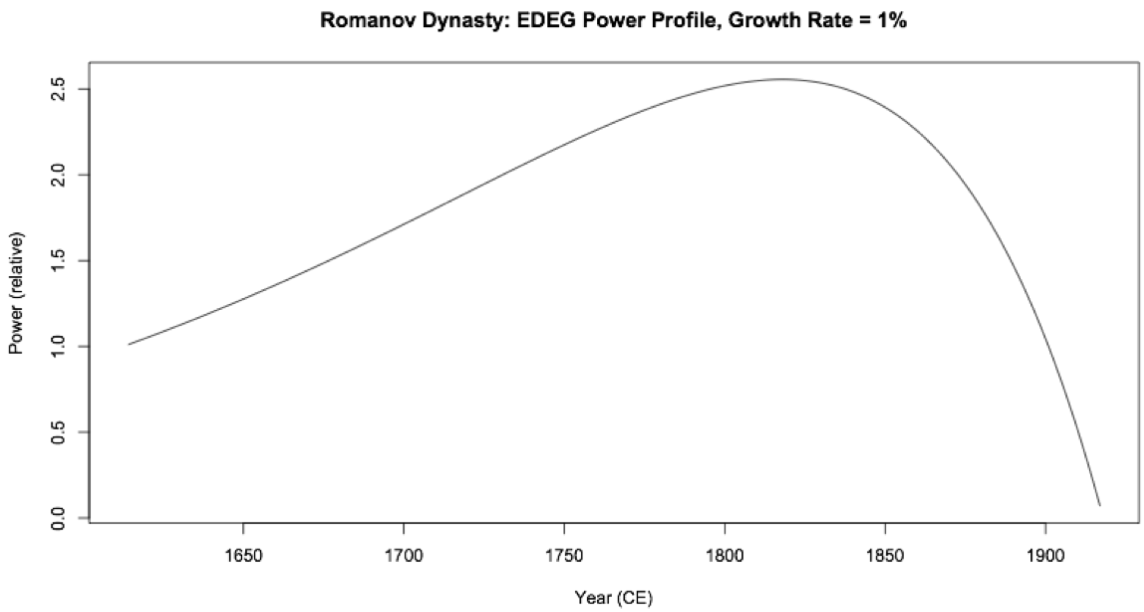

Figure 5. *Efficiency-discounted exponential growth (EDEG).*

Table 1. Summary of Analogous Concepts.

Entropy Approach	Emergy Approach	Comments
Nonrenewable Resource	Stock	May be semi-renewable
Potential	Stock + Consumer(s)	Not all stocks = potentials
Gibbs Free Energy	Emergy	Not identical
Exponential Growth	Growth in emergy	Not identical
Efficiency	Transformity	Not identical
Heat Engine	Producer	
Reproducing Heat Engine	Producer(s)+Consumer(s)	
Maximum entropy production	Maximum empower	Implications TBD

DISCUSSION AND FUTURE DIRECTIONS

The emergy approach holds the promise to expand the physical, entropic approach beyond the confines of closed systems. It also makes available a wealth of data. Similarities and differences between power and empower progressions need to be further explored. Table 1 offers an initial attempt to suggest analogues, but several are only loosely correlated.

The relation between efficiency, transformity, and quality of life should be further explored. Transformity per capita may differ from total dynastic transformity. A future goal would be to determine a transformity for natural resource cases such as the San Juans case where process types are well-documented. A valuable insight gained by an emergy perspective is that the availability of renewable resources considered in may determine key characteristics of a long-term society as contrasted to a medium-term dynasty.

The possibilities for further collaboration and exploration are tremendous, and they should provide great insights to both perspectives.

REFERENCES

Arto Annila and Stanley Salthe, 2010. Physical foundations of evolutionary theory. J. Non-Equilib. Thermodyn. 35: 301-321.

Bradley Carroll and Dale Ostlie, 2007. An Introduction to Modern Astrophysics, 2nd Ed. Pearson Addison-Wesley, San Francisco, 316 pp.

Michael Butler, 1996. Animal Cell Culture and Technology IRL Press at Oxford University Press, Oxford.

Center for Environmental Policy. Energy, Exergy and Thermodynamics presentation (https://cep.ees.ufl.edu/emergy/documents/presentations/ICEAM-2=Energy&Exergy.pdf last viewed April 24, 2018).

Mark Ciotola, 2001. Factors Affecting Calculation of L, Kingsley, S., R. Bhathal, ed.s, Conference Proceedings 4273 International Society for Optical Engineering (SPIE).

Ciotola, 2014. Efficiency Discounted Exponential Growth (EDEG) Approach to Modeling the Power Progression of a Historical Dynasty. ArXiv, submitted on 18 Nov 2014.

Ciotola, Effects Upon the Progression of a Society and Its Business Structures Due to Dependency Upon a Nonrenewable Resource: The San Juan Mining Region, Richard Robinson Business History Workshop 2016-Business and Environment in History: Dealing in Nature and Ecology, Pasts and Futures, Portland State University 28-30 April 2016.

Robert L. Heilbroner, 1980. The Worldly Philosophers 5th Ed. Touchstone (Simon and Schuster).

Clyde Henderson, 1926. Mining in Colorado: A History of Discovery, Development and Production, Professional Paper 138 U.S. Geological Survey, U.S. Department of the Interior. Washington.

Anatole Grigorevich Mazour and J. M. Peoples, 1975. Men and Nations, A World History, 3rd Ed. Harcourt, Brace, Jovanovich. New York.

Dennis Meadows, Donella Meadows, Jorgen Randers and William Behrens III, 1972. Limits to Growth, Club of Rome Universe Books. New York.

F. Marvin, 1936. Comte: The Founder of Sociology, Chapman and Hall. London, 11 pp.

Howard T. Odum, Mark T. Brown, Sherry Brandt-Williams, 2000. Handbook of Emergy Evaluation—A Compendium of Data for Emergy Computation Issued in a Series of Folios: Folio #1 Introduction and Global Budget. (Draft Version for Comment). Gainesville, FL.

Ilya Prigogine, 1967. Introduction to Thermodynamics of Irreversible Processes Wiley, 67. New York.

Fred Spier, 2009. Big History: The Emergence of an Interdisciplinary Science? World History Connected 6(3).

Duane A. Smith, 1982. Song of the Drill and Hammer: The Colorado San Juans, 1860-1914. Colorado School of Mines Press, Golden, Colorado.

Eric Twitty, 2010. Mining Resources of San Juan County. National Parks Service, U.S. Department of the Interior, Lafayette, CO.